ATTACHMENTA: Comments on Appendix H, "Risk Assessment Methodology"

PacifiCorp's Stochastic Analysis Model

Appendix H describes the Risk Assessment Methodology employed to evaluate risks associated with those variables classified as *Stochastic Risk* variables. These variables include fuel prices, electric prices, loads, and hydrogeneration. The model used to evaluate stochastic risk is a two-factor lognormal mean-reversion model. The two factors are the short-run and long-run variations in the variables, where the short-run variables revert to the trend (or expected value or mean) of the corresponding long-run variable. Both the short-run and long-run variables are assumed to follow a lognormal distribution.[1]

The specific discrete time representation of the model, as defined in equations 1 and 2 of Appendix H of the IRP (page 321), is[2]:

$$S_t = S_{t-1} + \left(L_t - L_{t-1}\right) + \alpha_t\left(L_{t-1} - S_{t-1}\right) + \sigma_t^S \varepsilon_t^S - \frac{Var\left(S_t\right)}{2} \tag{1}$$

$$L_t = L_{t-1} + \delta_t - \frac{\left(\sigma_t^L\right)^2}{2} + \sigma_t^L \varepsilon_t^L \tag{2}$$

where S and L represent the short and long run respectively (e.g., S_t is the short-

[1] If the random variable Y = lnX is normally distributed with mean μ and standard deviation σ, then X is said to have a lognormal distribution with mean $e^{\mu + \frac{1}{2}\sigma^2}$ and variance $e^{2\mu + 2\sigma^2} - e^{2\mu + \sigma^2}$. The lognormal distribution is a right-hand skewed distribution, which has a left-hand limit of zero: $X > 0$.

[2] For convenience, we drop the subscript n, which depicts the stochastic variables: n = fuel prices, electric prices, etc.

run value of the stochastic variable in period t), α is the mean reversion parameter or rate, δ is the long-run drift or growth parameter, σ is a volatility parameter, and ε is a stochastic error term.[3]

To implement the model, values are needed for the mean-reversion, drift, and volatility parameters: α, δ, and σ.[4] Values for the long-run parameters δ and σ^L are "assumed" to be 0.95 and 14.51% respectively.[5] The short-run parameters α and σ^S are estimated using an autoregressive (continuous time) process with a one period lag ("AR(1)"):

$$p_t - p_{t-1} = \left(1 - e^{-\alpha}\right)\left(\overline{p} - p_{t-1}\right) + v_t$$

$$or \qquad (3)$$

$$p_t = \left(1 - e^{-\alpha}\right)\overline{p} + e^{-\alpha} p_{t-1} + v_t$$

where "p" is the natural log of the short-run stochastic variable (e.g., p = ln(fuel price))[6], "e" is the exponential function, α is the mean-reversion parameter, and "v" is an error term.[7] For convenience, the AR1 model can be written as:

$$p_t = a + b\, p_{t-1} + v_t \qquad (4)$$

where the intercept is defined as $a = \left(1 - e^{-\alpha}\right)\overline{p}$ and the slope is defined as $b = e^{-\alpha}$.

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[3] Further details of the model can be found in Appendix H pages 321-322 of the IRP.

[4] Values for the error term ε are found by randomly drawing from a standard normal distribution using simulation techniques.

[5] See Appendix H, page 324, of the IRP for an explanation of these values.

[6] Since the variables are assumed to follow a lognormal distribution, the natural log of the variable will follow a normal distribution.

[7] In Appendix H, page 323, PacifiCorp uses the symbol ε to represent the error term of the AR1 process. This should not be confused with the error terms in Equations 1 and 2 on page 321 of Appendix H. To avoid confusion, we have used the symbol v for the error term of the AR1 process.

Under the right conditions estimates of "a" and "b" can be used to estimate the mean-reversion parameter α, and estimates of the error term "v" can be used to estimate σ^S.

Observations and Comments on PacifiCorp's Stochastic Analysis Model

Although PacifiCorp does not explain or justify the choice of the discrete time representation of the stochastic analysis model, equations 1 and 2 herein or equations 1 and 2 on page 321 of the IRP report, the model does have some intuitive appeal. For example, equation one representing the short-run variable can be rewritten as:

$$S_t = L_t - (1-\alpha)(L_{t-1} - S_{t-1}) + \left[\sigma_t^S \varepsilon_t^S - \frac{Var(S_t)}{2}\right] \qquad (5)$$

Equation (5) implies that the value of the short-run stochastic variable in period t is a function of three factors: the long-run value in period t; the weighted deviation of the short-run value from its long-run value in the prior period t-1; and the net volatility of the variable in period t. In other words, the magnitude of the short-run deviation from the long-run trend will depend on the magnitude of that deviation in the prior period plus some amount of inherent variance or volatility. However, assuming that this interpretation is correct does not explain or justify its use relative to other specifications that may have been used. Therefore, the Division recommends that the Commission order PacifiCorp to provide a clear explanation of its choice.

In equations 1 and 2 of Appendix H, the mean-reversion parameter α, the drift parameter δ, and the volatility parameter σ carry a time subscript (see equations 1&2 herein). This implies that each of these parameters changes from one period to the next. It is not clear from the presentation in Appendix H how this is to be achieved or why these parameters would change from period to period or what the impact of this would be on the overall risk analysis.

Therefore, the Division recommends that the Commission order PacifiCorp to provide an explanation of this aspect of the model specification and, if necessary, quantify its impact on the risk analysis and portfolio valuation.

Referring to the AR1 process (specified herein as equation 4 and specified on page 323 of Appendix H), PacifiCorp states, "For daily (weekly, or other discrete) time data, the above process was estimated with **OLS** regression as an autoregressive lag 1 period (or AR(1)) equation." (Emphasis added). The Division interprets this statement to mean that ordinary least squares ("OLS") was applied to the AR1 regression equation:

$$p_t = a + b\,p_{t-1} + v_t \tag{6}$$

This raises several potential problems in that, due to the presence of the lagged dependent variable (p_{t-1}) on the right-hand side, OLS may not be an appropriate technique to estimate this regression equation. To justify the use of OLS, the error term v_t is assumed to be identically and independently normally distributed with a mean of zero and variance σ^2. In this context, identically implies that the variance of v_t is the same for every time period t; independently implies that the errors are not related across periods:

$$v_t \sim N\left(0, \sigma^2\right) \quad \forall\, t \quad and \quad E\left(v_t v_j\right) = o \;\forall\, t \neq j \tag{7}$$

Additionally, it is assumed that the explanatory variable (in this case p_{t-1}) is independent of the error term (i.e., $E(p_{t-1}v_t) = 0$). If these assumptions are "true", then the Gauss-Markov theorem shows that the OLS estimates of "a" and "b" are, in a statistical sense, the best estimates to use.

However, one or more of these assumptions are typically violated when using lagged dependent variables. In particular, the lagged dependent variable and the error term may be correlated, or the error terms may be correlated across time or both. If the assumptions are not met in the model, then the OLS estimates will not be best.[8] In cases where the assumptions do not hold, there are known simple transformations of the model that lead to correct estimates of "a" and "b." Therefore, the Division recommends that the Commission order PacifiCorp to detail the procedure used in estimating the AR1 process and the justification for using that procedure (i.e., what statistical tests were used to test the assumptions of the OLS model). Additionally, assuming one or more of the OLS assumptions is violated, the Division recommends that the Commission order PacifiCorp to quantify the impact on the estimates of "a" and "b", the subsequent estimates of the mean-reversion parameter α, and the subsequent impact on the portfolio valuation.

There is another potential problem to consider in the estimation of the AR1 regression equation: economic time series data is often non-stationary. Under either the OLS or the transformation procedures, it is assumed that the data is stationary – the variance or covariance of the error terms is constant across time. When using economic time series data, however, it is often the case that the variance or covariance will change depending on which time interval is being considered. That this is a problem is implied by PacifiCorp's specification of the stochastic analysis model. In particular, the volatility parameters in equations 1 and 2 carry a time subscript (σ_t^S *and* σ_t^L). There are known tests for non-stationarity and appropriate estimation techniques if these tests indicate a problem. Therefore, the Division recommends that PacifiCorp justify its estimation procedure or quantify the impact of this problem on its portfolio valuation.

Given appropriate estimates of the AR1 regression equation, it is possible to derive an estimate of the mean-reversion parameter α. The AR1 regression

[8] In particular the OLS estimates maybe inconsistent, biased, or inefficient.

equation you will recall is specified as:

$$p_t = a + b\,p_{t-1} + v_t \tag{8}$$

where $b = e^{-\alpha}$. Given the appropriate estimate of b, say $\hat{b}$, it appears that the estimate of the mean-reversion parameter would be:

$$\hat{\alpha} = -\ln \hat{b} \tag{9}$$

However, on page 323 of the IRP report, PacifiCorp specifies the estimate as:

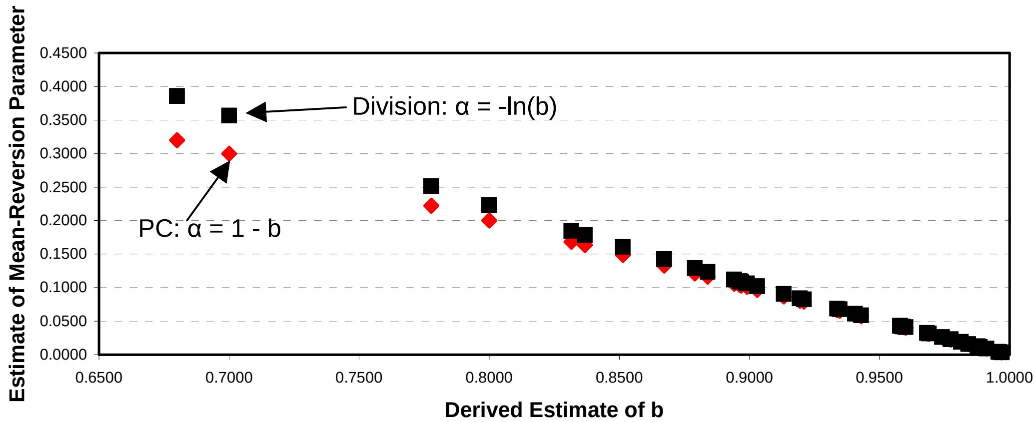
$$\hat{\alpha} = 1 - \hat{b} \tag{10}$$

It is not clear from the model's specification or subsequent discussion contained in Appendix H how this relationship is derived. However, in response to an informal data request, PacifiCorp explains that for small values of α, b ($= e^{-\alpha}$) is approximately equal to (1 – α) from which equation (10) follows.[9]

PacifiCorp's estimates of the mean-reversion parameter, α, are reported in Appendix H of the IRP report. These estimates can be used to back out the original estimates of b: $\hat{b} = 1 - \hat{\alpha}$. The derived estimates of b can then be used to re-estimate the mean-reversion parameter according to equation (9): $\hat{\alpha} = -\ln \hat{b}$. A comparison of the two alternative estimates of the mean-reversion parameter reveals that PacifiCorp's approximation (equation (10)) appears to systematically underestimate the mean reversion parameter. (See Figure 1 below).

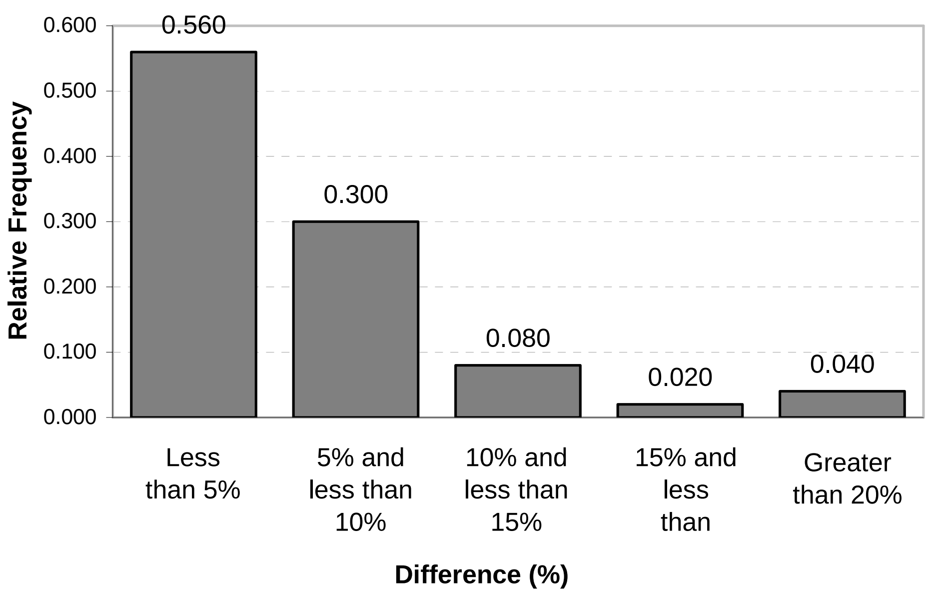
Figure 1: Comparison of Estimates of Alpha

[9] Jon Cassity, PacifiCorp employee, response to email inquiry, March 20, 2003.



On average, PacifiCorp's approximation of α is about 5% less than the alternative estimate. However, a few of the estimates are more than 20% less than the alternative specified above. (See Figure 2 below).

Figure 2: Distribution of Estimation Differences



Given the differences in the estimates of the mean-reversion parameter, the Division recommends that the Commission order PacifiCorp to justify its estimation of the mean-reversion parameter or quantify the impact of an incorrect estimation on its portfolio valuation.